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MULTIDIMENSIONAL APPROACH TO POVERTY MEASUREMENT: FUZZY MEASURES OF THE INCIDENCE AND THE DEPTH OF POVERTY*

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ABSTRACT

The paper presents a methodology for study of multi-dimensional aspects of poverty. In addition to the traditional uni-dimensional measures of poverty, exclusively estimated on the basis of the monetary variable (income or expenditure), other non-monetary variables are incorporated in analysis of the poverty phenomenon. The multidimensional approach has been based on the fuzzy sets theory in which the conventional poor/non-poor dichotomy is replaced by assessment of the degree of household poverty threat. The same methodology facilitates comprehensive monetary and non-monetary poverty analysis. In order to provide effective assessment of poverty the fuzzy measures of poverty were applied. The study employs the fuzzy measures to compare the degree of monetary poverty and deprivation threat in Polish voivodships in 2007 using micro data from the EU-SILC survey. The results of estimation of the fuzzy measures show that poverty in Poland has many dimensions and that its measurement solely from income standing vantage point is highly insufficient.

Key words: multi-dimensional poverty, fuzzy poverty measures, aggregation of indicators.

1. Introduction

The traditional approach to measuring poverty based on monetary indicators, whose foundations were set forth by the Material Welfare School (Marshall, 1920), dominated in nearly all research into this phenomenon up to the 1970s. In this approach the evaluation of the level of needs satisfaction was conducted exclusively on monetary variables (income or expenditure). Nevertheless, the

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viewpoint that the identification of impoverished persons exclusively on the basis of pecuniary categories is greatly insufficient gradually began to meet with considerable criticism. Nowadays, there is a widespread agreement that poverty is a multidimensional phenomenon and cannot be reduced solely to monetary dimension but has to be also explained by diverse non-monetary variables.

Many researchers have postulated the necessity of treating poverty multidimensionally. Townsend was one of the first persons to single out the imperfection inherent in identifying poverty exclusively on the basis of the income criterion. He proposed for poverty analyses to incorporate dwelling conditions, affluence, education as well as professional and financial resources (Abel-Smith and Townsend, 1973; Townsend, 1979). A broader look at the problem of poverty than just through the prism of income (expenditures) was also presented, among others, by Atkinson and Bourguignon (1982), Hagenaars (1986), Sen (1992), Panek (1996), Bourguignon and Chakravarty (2003), Tsui (2002), and Deuch and Silber (2005).

In this paper the multidimensional approach to poverty measurement based on application of the fuzzy set theory is presented. Moreover, the incidence and the depth of monetary and non-monetary poverty (deprivation) in Poland in 2007 by voivodships is analysed.

2. Characterization of the data

The bases for the analyses conducted are the data from the European Union Survey on Income and Living Conditions (EU-SILC) carried out by the Central Statistical Office (CSO, 2009). The main objective of EU-SILC is to supply EU comparable data on the living conditions of the population of the EU Members States. It is an instrument aiming at collecting timely and comparable cross-sectional and longitudinal microdata on income, poverty, social exclusion and living conditions. In order to satisfy these needs EU- SILC is carried out with the use of the panel method in the four-year cycle. The households were selected for EU-SILC survey in Poland by means of two-stage stratified sampling scheme with different selection probabilities at the first stage. Before selection sampling units were stratified by voivodship, and then, within voivodships, according to the class of locality (rural areas and urban areas grouped by size of town). The first-stage sampling units (primary sampling units-PSU's) were census areas. During the second stage, dwellings were drawn systematically from a randomly generated list of dwellings, independently within each stratum created during the first stage. All the households, including all household members, from the selected dwellings entered the sample.

In the empirical results presented in the article, the data for Poland for 2007 were applied. In 2007, 14286 households were examined and individually 34898 members of these households age 16 or more. In order to generalize the results

over the entire population the appropriate system of weights was applied, which takes into consideration:

- selection probability for dwellings and households,
- survey completeness according to the class of locality,
- consistency of the composition of the household and individual populations according to age and gender, with the external demographic data available.

Estimation for standard errors was based on resampling approach. It used a bootstrap method with resamples 500 times from each stratum n_h-1 PSU's with replacement (McCarthy and Snowden, 1985), where n_h denotes the sample of PSU in the n-th strata.

3. Assumptions of the poverty analysis

In order to provide effective assessment of poverty the fuzzy incidence of relative poverty (monetary poverty), the fuzzy incidence of relative deprivation (non-monetary poverty) indicators, the fuzzy depth of relative poverty and the fuzzy depth of relative deprivation indicators were applied. The fuzzy incidence indicators are counterparts of the headcount ratio in the traditional approach defined as:

$$H = \frac{q}{n}, \tag{1}$$

where: q – the number of poor.

The fuzzy depth indicators are counterparts of the income gap index in conventional approach which is defined as follows:

$$I = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z - y_i}{z}\right), \tag{2}$$

where:

y_i – equivalized income of the i-th individual,

z – poverty line,

n – the number of individuals (households or persons).

The construction of fuzzy measures bases on the fuzzy set approach introduced by Cerioli and Zani (1990) who drew inspiration from the theory of Fuzzy Sets initiated by Zadeh (1965). Cerioli and Zani's proposal was developed by Cheli and Lemmi (1995), and further has been followed by a number of applications (*Fuzzy Set Approach...*, 2006).

The fuzzy set approach makes it possible to avoid simple dichotomization of the population under research into poor and non-poor defined in the relation to some chosen treshold border (poverty line). Poverty is not defined in terms of presence or absence in the subset of poor individuals but as a matter of degree of

belonging to this sub-set. The *i*-th individuals' propensities to poverty is measured by the so-called *membership function* (*m.f.*) to the poverty sphere (μ_i). It assumes value 1 when an individual belongs to poverty set completely, value 0 when an individual does not belong to poverty set, and values between 0 and 1 when it belongs to poverty set partially. Its degree of membership of poverty set increases in proportion to the proximity of *m.f.* to 1.

4. Fuzzy Incidence Indicators

4.1. Fuzzy Monetary Incidence (FMI) Indicator

Cerioli and Zani defined the *m.f.* for the monetary dimension of poverty introducing two threshold values: an individual is definitely poor when he or she is below the first one and when he or she is above the second one . In the latter case, after crossing the threshold, the *m.f.* declines from 1 to 0 linearly.

In order to avoid arbitrary selection of threshold values Chelli and Lemmi (1995) defined the *m.f.* as the distribution function $F(y_i)$ of income, normalised (linearly transformed) so as to equal 1 for the poorest and 0 for the richest individual in the population. Betti and Verma (1999) modified the *m.f.* version proposed by Chelli and Lemmi taking the *m.f.* as the normalised (linearly transformed) Lorenz curve (function) of income $L(F(y_i))$. It varies from 1 for the poorest to 0 for the richest individual. Finally, Betti, Chelli, Lemmi and Verma (2006) combined their previous proposals into the *Integrated Fuzzy and Relative* (IFR) approach.

The Fuzzy Monetary Incidence (FMI) indicator defined under the IFR approach (serving simultaneously as the definition of *m.f.*), combines the (1- $F_{(MI)}$) indicator proposed by Chelli and Lemmi and the (1-$L_{(MI)}$) indicator proposed by Betti and Verma. The (1-$F_{(MI)}$) indicator is the proportion of individuals who are less poor then the individual concerned (their degree of poverty is less marked than the individual concerned):

$$\mu_i(y) = FMI_i = \left(1 - F_{(MI),i}\right)^{\alpha} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma} \right)^{\alpha}, \; i=1,2,...,n; \; \mu_n(y)=0, \qquad (3)$$

where:

y_i – equivalised income of the *i*-th individual,

$F_{(MI),i}$ – value of the income distribution function $F(y_i)$ for the *i*-th individual,

w_γ – weight of the *i*-th individual of rank γ in the ascending income distribution,

α – parameter.

The (1-$L_{(MI)}$) indicator is the share of total equivalised income received by all individuals who are not as poor as the person concerned:

$$\mu_i(y) = FMI_i = \left(1 - L_{(MI),i}\right)^{\alpha} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma y_\gamma}{\sum_{\gamma=2}^{n} w_\gamma y_\gamma} \right)^{\alpha}, \; i=1,2,...,n; \; \mu_n(y)=0, \qquad (4)$$

where:

$L_{(MI),i}$ – the value of the Lorenz curve of income $L(F(y_i)$ for the i-th individual.

Finally, the FMI indicator (the *m.f.*) is defined as a combination of the previous forms (3) and (4):

$$\mu_i(y) = FMI_i = \left(1 - F_{(MI),i}\right)^{\alpha-1}\left(1 - L_{(MI),i}\right), \; i=1,2,...,n. \qquad (5)$$

The parameter α is estimated so that the mean of the FMI indicator (the mean of *m.f.*) is equal to the head count ratio (1) computed for the official poverty line.

The FMI indicator (the incidence of relative poverty indicator) for the population takes the following form:

$$FMI = \frac{\sum_{i=1}^{n} FMI_i \cdot w_i}{\sum w_i}. \qquad (6)$$

5. Fuzzy Supplementary Incidence Indicator (FSI)

In addition to the monetary (income) variable, poverty in the multidimensional approach is also explained by non-monetary variables describing the standard of living of households and individuals. The starting point for including non-monetary variables in poverty analysis is the selection of variables that may be treated as deprivation symptoms (z_j; j=1,2,...,k_h) (to be included in the index or indices of deprivation dimensions) and grouping them into deprivation dimensions (h; h=1,2,...,m). Deprivation symptoms (variables) may take the form of dichotomous[1] or polychotomous variables[2]. The next step is to assign numerical values to each deprivation symptom ordered categories. Then, it is necessary to weight the deprivation symptoms scores in order to construct composite indicators and to scale the measures.

[1] The absence of certain goods or facilities due to financial reasons, for example a car or warm running water.

[2] For example, home mortgage loan defaults (from the absence of default, to default by one month to default of more than six months).

In the calculation of the FSI indicator procedure, numerical values (ranks) are assigned to each deprivation symptom category (c=1,2,...,u) after arranging the deprivation symptom categories from the most deprived (c=1) to the least deprived (c=u) situation[1]. Then, for each deprivation symptom we determine a quantitative deprivation score (assessment of the degree of deprivation) using the following formula:

$$e_{hj,i} = \frac{1 - F\left(c_{hj,i}\right)}{1 - F(1)}, \quad h=1,2,...,m; \ j=1,2,...,k_h; \ i=1,2,...,n, \tag{7}$$

where:

$c_{hj,i}$ – value of the category of the j-th deprivation symptom in the h-th dimension for the i-th individual,

$F(c_{hj,i})$ – value of the j-th deprivation symptom distribution function in the h-th dimension for the i-th individual.

The formula (7) is identical for dichotomous and polychotomous variables (deprivation symptoms).

Using a system of weights, an overall non-deprivation score indicating lack of deprivation, for the i-th individual and for each deprivation dimension separately, is determined:

$$e_{h,i} = \frac{\sum_{j=1}^{k_h} w_{hj}\left(1 - e_{hj,i}\right)}{\sum_{j=1}^{k_h} w_{hj}}, \quad h=1,2,...,m; \ i=1,2,...,m, \tag{8}$$

where:

w_{hj} – weight of the j-th deprivation symptom in the h-th dimension.

Weights are calculated separately within each dimension. The weighting procedure is based on a statistical consideration taking into account the dispersion of deprivation symptoms and its correlation with other deprivation symptoms in the given dimension (see also: Gianni and Verma, 2008). The weights to be given to deprivation indicators (scores) are calculated according to the following formula:

$$w_{hj} = w_{hj}^{a} \cdot w_{hj}^{b}, \quad h=1,2,...,m; \ j=1,2,...,k_h, \tag{9}$$

where:

w_{hj}^{a} – measure of dispersion of the j-th deprivation symptom in the h-th dimension,

[1] For example, for the absence of a car c=1 and for the possession of a car c=2, home mortgage loan defaults from more than sixth months (c=1) to the absence of default (c=u).

w_{hj}^{b} – measure of correlation of the *j*-th deprivation symptom in the *h*-th dimension with other deprivation symptoms in this dimension.

A measure of dispersion is defined as follows:

$$w_{hj}^{a} = V^{(k)}\left(e_{hj}\right) = \frac{S\left(e_{hj}\right)}{\overline{e}_{hj}},\ j=1,2,...,\ k_h;\ h=1,2,...,m, \tag{10}$$

where:

$S\left(e_{hj}\right)$ – standard deviation of deprivation score for the *j*-th deprivation symptom in the *h*-th dimension,

$\overline{e}_{hj}$ – arithmetic mean of deprivation score for the *j*-th deprivation symptom in the *h*-th dimension.

A measure of correlation is defined according to the following form:

$$w_{hj}^{b} = \left(\frac{1}{1+\sum_{j=1}^{k_h}\left|r_{e_{hj,hj'}}\right|\left|r_{e_{hj,hj'}} < r_{e_{hj}}^{*}\right.}\right)\left(\frac{1}{\sum_{j=1}^{k_h}\left|r_{e_{hj,hj'}}\right|\left|r_{e_{hj,hj'}} \geq r_{e_{hj}}^{*}\right.}\right),\ j,j'=1,2,...,k_h, \tag{11}$$

where:

$r_{e_{hj,hj'}}$ – correlation coefficient between deprivation indicators corresponding to the *j*-th and the *j'*-th deprivation symptoms in the *h*-dimension,

$r_{e_{hj}}^{*}$ – critical value (threshold) of the correlation coefficient of the *j*-th deprivation symptom in the *h*-th dimension.

The critical value of the correlation coefficient $r_{e_{hj}}^{*}$ is calculated on the basis of the set of absolute values of correlation coefficients between deprivation indicators (corresponding to the *j*-th and *j'*-th deprivation symptoms in the *h*-dimension) arranged in ascending order extended by including 0 and 1:

$$A = \{0, \left|r_{e_{hj,hj'}}^{(1)}\right|, ..., \left|r_{e_{hj,hj'}}^{(k_h-1)}\right|, 1\}.\ j,j'=1,2,...,k_h;\ j'\neq j. \tag{12}$$

In the next step an overall non-deprivation indicator (non-deprivation score) for any individual is calculated. Since we assume that all deprivation dimensions are of identical importance, they should be equally weighted using the following formula:

$$e_i = \frac{\sum_{h=1}^{m} e_{h,i}}{m},\ i=1,2,...,n. \tag{13}$$

As in the calculation of the FMI indicator, the overall FSI indicator (the *m.f.*), for the *i*-th individual, is defined as a combination of the (1-$F_{(SI),i}$) indicator and the (1-$L_{(SI),i}$) indicator:

$$\mu_i(z) = FSI_i = \left(1 - F_{(SI),i}\right)^{\alpha'-1}\left(1 - L_{(SI),i}\right),\ i=1,2,...,n. \tag{14}$$

The $\left(1 - F_{(SI),i}\right)$ indicator, for the *i*-th individual, is the proportion of individuals who are less deprived than the individual concerned (their degree of deprivation is lower than for the individual concerned):

$$\mu_i(z) = FSI_i = \left(1 - F_{(SI),i}\right)^{\alpha'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma}\right)^{\alpha'},\ i=1,2,...,n;\ \mu_n(z)=0, \tag{15}$$

where:

$F_{(SI),i}$ – value of the non-deprivation score distribution function $F(e_i)$ for the *i*-th individual,

w_γ – weight of the *i*-th individual of rank γ in ascending non-deprivation score distribution,

α' – parameter.

The (1-$L_{(SI),i}$) indicator is the share of the total non-deprivation score assigned to all individuals less deprived than the person concerned (their deprivation risk is lower than for the individual concerned):

$$\mu_i(z) = FSI_i = \left(1 - L_{(SI),i}\right)^{\alpha'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma e_\gamma}{\sum_{\gamma=2}^{n} w_\gamma e_\gamma}\right)^{\alpha'},\ i=1,2,...,n;\ \mu_n(z)=0, \tag{16}$$

where:

$L_{(SI),i}$ – value of the Lorenz curve of the non-deprivation score $L\left(F\left(e_i\right)\right)$ for the *i*-th individual.

The overall (for the population in question) Fuzzy Supplementary Incidence indicator (the incidence of relative deprivation indicator) is defined as:

$$FSI = \frac{\sum_{i=1}^{n} FSI_i w_i}{\sum_{i=1}^{n} w_i}. \tag{17}$$

As for the FMI indicator, the parameter α' in equation (14) is determined so as to make the overall FSI indicator equal to the head count ratio (1). The parameter α' is also used to calculate the FSI indicator for every single dimension.

The FSI indicator, in the *h*-th dimension, for the *i*-th individual is defined as:

$$\mu_i(z) = FSI_{hi} = \left(1 - F_{(SI),hi}\right)^{\alpha'-1}\left(1 - L_{(SI),hi}\right),\ h=1,2,...,m;\ i=1,2,...,n. \qquad (18)$$

Finally, the overall Fuzzy Supplementary Incidence indicator (the incidence of relative deprivation indicator) for the *h*-th dimension for the population is calculated as the following mean:

$$FSI_h = \frac{\sum_{i=1}^{n} FSI_{h,i} \cdot w_i}{\sum_{i=1}^{n} w_i};\ h=1,2,...,m. \qquad (19)$$

Fuzzy Depth Indicators

Fuzzy incidence indicators defined under the FR approach overlook the second basic aspect of poverty analysis, namely poverty depth. The necessity of also taking poverty depth into consideration in multidimensional analyses of poverty has been postulated by many researchers (see, for example Shorrocks and Subramanian, 1994). Panek (2009) proposed to extend the IFR approach by incorporating two additional indicators, namely the Fuzzy Monetary Depth indicator (FMD) and the Fuzzy Monetary Supplementary Depth indicator (FSD).

6. Fuzzy Monetary Depth Indicator (FMD)

The starting point for defining the FMD indicator is the calculation of the income (poverty) gap ratio for each individual:

$$v_i = \frac{z - y_i}{z},\ i=1,2,...,n, \qquad (20)$$

with the non-poor individuals v_i being assigned the value of zero.

In the next step, we define the degree of the lack of poverty gap (non-poverty gap score) for each individual:

$$d_i = 1 - v_i,\ i=1,2,...,n. \qquad (21)$$

d_i is a positive score indicating a lack of poverty gap and is analogous to y_i in the construction of the FMI indicator.

The FMD indicator is defined, similarly to the FMI indicator, as a combination of the $(1\text{-}F_{(MD)})$ indicator and the $(1\text{-}L_{(MD)})$ indicator.

The $(1\text{-}F_{(MD),i})$ indicator for the i-th individual is the proportion of individuals whose non-poverty gap score is higher (who are not as poor or better off) than the individual concerned:

$$\mu_i(d) = FMD_i = \left(1 - F_{(MD),i}\right)^{\beta} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma} \right)^{\beta}, \quad i=1,2,...,n; \ \mu_n(d)=0, \qquad (22)$$

where:

$F_{(MD),i}$ – value of the distribution function $F(d_i)$ of the non-poverty gap score for the i-th individual,

w_γ – weight of the i-th individual of rank γ in ascending non-poverty gap score distribution,

β – parameter.

The $(1\text{-}L_{(MD),i})$ indicator is the share of the total non-poverty gap score assigned to all individuals whose non-poverty gap score is higher (who are not as poor or are better off) than the individual concerned:

$$\mu_i(d) = FMD_i = \left(1 - L_{(MD),i}\right)^{\beta} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma d_\gamma}{\sum_{\gamma=2}^{n} w_\gamma d_\gamma} \right)^{\beta}, \quad i=1,2,...,n; \ \mu_n(d)=0, \quad (23)$$

where: $L_{(MD),i}$ – value of the Lorenz curve of the non-poverty gap score $L\left(F\left(d_i\right)\right)$ for the i-th person.

Finally, the degree of poverty gap, for the i-th individual, is defined as a combination of formulas (22) and (23):

$$\mu_i(d) = FMD_i = \left(1 - F_{(MD),i}\right)^{\beta-1}\left(1 - L_{(MD),i}\right), \ i=1,2,...,n. \qquad (24)$$

The overall (for the population in question) Fuzzy Monetary Depth indicator (the depth of relative deprivation indicator) is calculated as follows:

$$FMD = \frac{\sum_{i=1}^{n} \mu_i(d) \cdot w_i}{\sum_{i=1}^{n} w_i}. \qquad (25)$$

The parameter β in equation (24) is estimated so that the mean of the FMD indicator (for the entire population) is equal to the poverty gap index (2).

7. Fuzzy Supplementary Depth indicator (FSD)

The starting point for calculating the FSD indicator is the same set of deprivation symptoms as was established for the FSI indicator. Then, we determine a quantitative deprivation gap ratio for each deprivation symptom, and for each individual:

$$x_{hj,i} = \frac{\left(c_{hj} = r-1\right) - \left(e_{hj,i} - 1\right)}{\left(c_{hj} = r-1\right)}, \quad h=1,2,...,m; j=1,2,...,k_n; i=1,2,...,n, \qquad (26)$$

with the non-deprived individuals, with regard to the j-th symptom in the h-dimension, $x_{hj,i}$ being set to zero (for individual, for which rank assumes value $c_{hj,i} \geq r$; $c_{nj}=1,2,...,u$; r≤u), where:

$c_{hj}=r$ – rank of the j-th deprivation symptom category in the h-th dimension for which deprivation is not found.

The above formula is identical for dichotomous and polychotomous variables (deprivation symptoms).

In the next step the degree of the lack of deprivation gap (non-deprivation gap score) for each individual is calculated:

$$s_{hj,i} = 1 - x_{hj,i}, \; h=1,2,...,m; j=1,2,...,k_n; i=1,2,...,n. \qquad (27)$$

Then, we determine the deprivation gap score (assessment of the degree of deprivation gap) for each deprivation symptom:

$$g_{hj,i} = \frac{1 - F\left(s_{hj,i}\right)}{1 - F\left(1\right)}, \; h=1,2,...,m; j=1,2,...,k_h; i=1,2,...,n, \qquad (28)$$

where: $F(s_{hj,i})$ – value of the distribution function of the non-deprivation gap score, regarding the j-th deprivation symptom in the h-th dimension, for the i-th individual.

Using the system of weights, the same that was applied in the calculation of FMI indicator, the non-deprivation gap score for the i-th individual, and for each deprivation dimension separately, is determined:

$$g_{h,i} = \frac{\sum_{j=1}^{k_h} w_{hj}\left(1 - g_{hj,i}\right)}{\sum_{j=1}^{k_h} w_{hj}}, \; h=1,2,...,m; i=1,2,...,n. \qquad (29)$$

In the next step the non-deprivation gap scores (29) are aggregated into the overall deprivation gap score indicating lack of deprivation gap, for the each i-th person, as the unweighted mean:

$$g_i = \frac{\sum_{h=1}^{m} g_{h,i}}{m},\ i=1,2,...,n. \tag{30}$$

The FSD indicator, for the *i*-th individual, is calculated as a combination of the (1-$F_{(SD,i)}$) indicator and the (1-$L_{(SD),i}$) indicator:

$$\mu_i(s) = FSD_i = \left(1 - F_{(SD),i}\right)^{\beta'-1}\left(1 - L_{(SD),i}\right),\ \text{i=1,2,...,n.} \tag{31}$$

The (1-$F_{(SD,i)}$) indicator, for the *i*-th individual, is the proportion of individuals non-deprivation gap score is higher (which are less deprived) than the individual concerned:

$$\mu_i(s) = FSD_i = \left(1 - F_{(SD),i}\right)^{\beta'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma}{\sum_{\gamma=2}^{n} w_\gamma}\right)^{\beta'},\ i=1,2,...,n;\ \mu_n(s)=0, \tag{32}$$

where:

$F_{(SD),i}$ – value of the distribution function $F(g_i)$ of the lack of the deprivation gap score for the *i*-th individual,

w_γ – weight of the *i*-th individual of rank γ in the ascending lack of the deprivation gap score distribution,

β' – parameter.

The (1-$L_{(SD),i}$) indicator, for the *i*-th individual, is the share of the total non-deprivation gap score assigned to all individuals whose non-deprivation gap score is higher than the individual concerned:

$$\mu_i(s) = FSD_i = \left(1 - L_{(SD),i}\right)^{\beta'} = \left(\frac{\sum_{\gamma=i+1}^{n} w_\gamma g_\gamma}{\sum_{\gamma=2}^{n} w_\gamma g_\gamma}\right)^{\beta'},\ i=1,2,...,n;\ \mu_n(s)=0, \tag{33}$$

where: $L_{(SD),i}$ – value of the Lorenz curve of the non-deprivation gap score for the *i*-th individual.

Finally, the Fuzzy Supplementary Depth indicator (the depth of relative deprivation indicator) for the population is defined as the following mean:

$$FSD = \frac{\sum_{i=1}^{n} \mu_i(s) \cdot w_i}{\sum_{i=1}^{n} w_i}, \tag{34}$$

The parameter β' is estimated so that the FSD indicator for the population is equal to the income gap index defined by (2).

The FSD indicator for the *i*-th individual and for the each *h*-th deprivation dimension is calculated as follows:

$$\mu_i(s_h) = FSD_{h,i} = \left(1 - F_{(SD)h,i}\right)^{\beta'-1}\left(1 - L_{(SD)h,i}\right),\ h=1,2,...,m;\ i=1,2,...,n. \quad (35)$$

Finally, the Fuzzy Supplementary Depth indicators for each *h*-th deprivation dimension for the population are defined as:

$$FSD_h = \frac{\sum_{i=1}^{n} \mu_i(s_h) \cdot w_i}{\sum_{i=1}^{n} w_i},\ h=1,2,...,m. \quad (36)$$

Empirical results

The fuzzy poverty (monetary) measures are based on the household equivalised income variable. The household equivalised income is defined as the household disposable income divided by the household equivalence scale. The household disposable income includes net monetary incomes gained by all the household members reduced by property tax, inter-household cash transfers paid and statements for the Treasury Office. The equivalence scales are the parameters which allow for comparing the income of households of various sizes and demographic composition. We have used the modified OECD equivalence scale, which assigns a weight of 1 to the first adult household member, 0.5 to every other adult household member, and 0.3 to every child in the household under 14.

The poverty line applied for calculation traditional poverty indicators (1) and (2) is that of 60% of median equivalised disposable income after social transfers.

To measure the incidence and the depth of deprivation (non-monetary poverty) two initial steps, as it was mentioned, were necessary. At first, using the rich EU-SILC data, a subset of substantively meaningful and useful indicators (deprivation symptoms) for deprivation analysis were selected. Next, applying factor analysis, they were grouped into five dimensions (Whelaan *et al*., 2001). The final list of the selected deprivation dimensions and deprivation symptoms is presented in the Table 1[1]. All these indicators are considered at the household level and subsequently are assigned to the household members.

[1] The selection of deprivation symptoms and grouping them into deprivation dimension has been done by Caterina Ferreti from CRIDIRE, University of Siena (Italy).

Table 1. Dimensions and symptoms of deprivation

No.	Dimensions and indicators
1	**Basic life style – symptoms relate to the lack of ability to afford most basic requirements:**
1.1	Paying for one week annual holiday away home
1.2	Eating meal with meat, chicken, fish (or vegetarian equivalent) every second day
1.3	Keeping home adequately warm
1.4	Ability to make ends meet
2	**Household arrears and unexpected financial expenses - symptoms relate to arrears that the household has experienced and to unexpected financial expenses it faced in the last 12 months:**
2.1	Arrears on mortgage or rent payments
2.2	Arrears on utility bills
2.3	Arrears on hire purchase instalments or other loan payments
2.4	Facing unexpected financial expenses
3	**Housing facilities and deterioration – symptoms relate to the absence of basic housing facilities and to serious problems with the dwelling:**
3.1	A bath or shower in dwelling
3.2	An indoor flushing toilet for sole use of household
3.3	Leaky roof, damp walls/floors/foundation, or rot in window frames or floor
3.4	Too dark, not enough light in dwelling
4	**Environmental problems – symptoms relate to the neighbourhood and the environment:**
4.1	Noise from neighbours or from the street
4.2	Pollution, grime or other environmental problems
4.3	Crime violence or vandalism in the area
5	**Equipment of households in durables – symptoms relate to the lack of possession of a widely – desired durables because of lack of resources:**
5.1	A telephone (including mobile phone)
5.2	A colour TV
5.3	A computer
5.4	A washing machine
5.5	A car

Table 2. Fuzzy Incidence Indicators in Poland by voivodships in 2007

Voivodships	Indicator values in percentages						
	FMI	FSI	$FSI_{h=1}$	$FSI_{h=2}$	$FSI_{h=3}$	$FSI_{h=4}$	$FSI_{h=5}$
Dolnośląskie	17.7	20.0	21.8	13.0	15.3	20.2	13.8
	6.2*	5.6*	6.2*	7.4*	6.3*	6.2*	6.5*
Kujawsko-pomorskie	17.8	16.7	20.4	11.7	11.8	15.4	12.7
	7.3*	8.3*	9.3*	10.6*	11.9*	8.7*	9.2*
Lubelskie	24.2	19.0	23.1	12.8	17.0	10.5	14.3
	5.6*	7.2*	6.8*	9.9*	8.7*	10.6*	7.5*
Lubuskie	16.7	17.4	28.6	9.9	11.1	10.4	12.6
	10.4*	10.5*	9.3*	18.0*	13.0*	17.8*	11.6*
Łódzkie	17.6	20.8	24.6	11.6	17.8	16.6	13.6
	5.7*	6.0*	5.7*	8.6*	7.0*	7.9*	6.1*
Małopolskie	17.4	18.9	26.5	10.8	10.9	15.8	13.9
	5.9*	5.8*	5.9*	8.9*	8.0*	8.0*	6.6*
Mazowieckie	14.8	15.2	17.8	9.7	14.2	14.4	11.3
	4.9*	5.1*	5.4*	7.0*	5.6*	5.8*	5.5*
Opolskie	14.2	14.6	18.4	8.6	10.5	15.1	10.6
	14.2*	13.8*	12.7*	16.9*	17.6*	16.3*	14.3*
Podkarpackie	23.5	18.6	28.4	11.5	11.9	10.5	12.8
	6.4*	7.7*	6.6*	9.5*	8.9*	12.5*	7.5*
Podlaskie	17.7	11.7	16.8	9.9	11.2	7.9	12.9
	12.8*	12.9*	11.3*	16.5*	19.7*	16.7*	13.2*
Pomorskie	16.6	18.6	21.2	10.9	13.5	17.8	13.0
	8.0*	7.0*	7.6*	13.3*	9.2*	8.6*	8.0*
Śląskie	13.5	17.3	21.7	10.3	10.0	18.7	10.3
	5.2*	5.0*	4.6*	6.9*	6.6*	5.4*	6.2*
Świętokrzyskie	20.9	17.9	28.1	9.1	13.8	10.2	11.0
	7.8*	8.5*	8.2*	13.7*	11.3*	13.6*	12.0*
Warmińsko-mazurskie	21.5	18.8	26.0	9.1	14.2	12.2	12.5
	8.6*	8.0*	7.6*	14.9*	11.2*	14.7*	11.8*
Wielkopolskie	16.3	14.1	19.3	10.5	10.7	13.8	11.5
	6.1*	6.7*	6.7*	9.4*	9.7*	8.58	7.4*
Zachodniopomorskie	16.4	17.3	23.9	10.9	11.1	13.7	11.2
	8.4*	8.2*	8.5*	10.6*	11.2*	11.1*	10.5*
Poland	17.3	17.3	22.3	10.8	12.8	14.8	12.3
	2.2*	2.2*	2.0*	2.2*	2.4*	2.0*	2.1*

**Relative standard error · 100.*

Source: Central Statistical Office, EU-SILC Survey data, wave 3, version dated of 01.08.2009. Survey co-financed by UE. The views expressed are solely those of the author and should not be attributed to the European Commission.

Table 3. Fuzzy Depth Indicators in Poland by voivodships in 2007

Voivodships	Indicator values in percentages						
	FMD	FSD	$FSD_{h=1}$	$FSD_{h=2}$	$FSD_{h=3}$	$FSD_{h=4}$	$FSD_{h=5}$
Dolnośląskie	6.3	6.5	10.5	4.0	5.8	8.7	4.0
	10.9*	10.7*	10.3*	13.7*	9.9*	11.3*	12.1*
Kujawsko-pomorskie	4.7	4.2	8.2	3.8	5.1	5.8	3.8
	16.1*	17.5	13.8*	17.1*	18.2*	15.3*	18.4*
Lubelskie	8.0	5.3	10.0	4.4	7.5	4.6	3.8
	11.0*	13.1*	11.5*	15.4*	14.7*	17.4*	12.6*
Lubuskie	3.8	4.4	15.5	2.8	3.5	4.0	2.7
	20.9*	23.2*	15.3*	30.2*	21.5*	36.6*	20.1*
Łódzkie	5.2	7.8	12.2	4.3	8.9	8.0	3.8
	12.7*	11.0*	9.8*	14.5*	10.7*	12.3*	13.5*
Małopolskie	4.7	5.2	13.3	3.3	3.9	6.7	4.6
	13.4*	10.9*	9.1*	15.8*	12.8*	13.7*	11.6*
Mazowieckie	4.0	4.6	8.6	3.6	6.2	5.7	3.2
	10.0*	10.6*	8.8*	13.8*	9.6*	10.9*	9.7*
Opolskie	4.3	3.1	6.4	2.1	2.8	6.6	2.8
	27.6*	23.6*	22.2*	30.0*	24.6*	26.6*	22.7*
Podkarpackie	6.8	4.8	12.8	3.2	4.5	3.4	2.9
	12.7*	14.6*	11.7*	19.8*	13.8*	21.8*	14.3*
Podlaskie	4.6	2.6	6.5	2.8	4.7	2.5	4.0
	30.9*	37.4*	25.7*	24.1*	27.5*	25.3*	28.0*
Pomorskie	5.0	5.7	10.5	3.4	4.5	8.2	3.7
	14.5*	14.0*	13.0*	20.0*	14.8*	13.3*	17.1*
Śląskie	4.1	5.7	10.5	3.7	3.4	9.4	3.0
	9.9*	9.0*	7.1*	11.7*	10.3*	8.9*	11.1*
Świętokrzyskie	5.8	4.7	14.5	2.2	7.1	2.6	2.7
	15.4*	16.9*	14.3*	23.4*	16.9*	21.4*	20.4*
Warmińsko-mazurskie	6.7	4.7	11.9	2.9	5.7	5.6	2.4
	16.2*	16.0*	13.4*	19.7*	16.6*	20.6*	20.1*
Wielkopolskie	4.3	3.5	8.7	3.4	4.1	5.1	2.5
	13.4*	15.0*	13.0*	16.6*	18.2*	14.8*	14.3*
Zachodniopomorskie	4.3	4.5	12.5	3.2	4.1	6.2	2.8
	16.3*	16.5*	13.7*	20.1*	17.2*	18.5*	17.0*
Poland	5.0	5.0	10.6	3.5	5.2	6.3	3.4
	3.2*	3.2*	3.1*	3.4*	3.5*	3.6*	3.3*

**Relative standard error · 100.*

Source: Central Statistical Office, EU-SILC Survey data, wave 3, version dated of 01.08.2009. Survey co-financed by UE.

Fuzzy poverty incidence in Poland by voivodships

Voivodships of the greatest fuzzy poverty incidence (Table 2) were in 2007 Lubelskie, Podkarpackie, Warmińsko-mazurskie and Świętokrzyskie voivodships, and those with the lowest were Śląskie, Opolskie and Mazowieckie. The hierarchy of voivodships according to the fuzzy deprivation incidence is different than according to the fuzzy poverty incidence. The highest level of fuzzy deprivation incidence was observed in Dolnośląskie and Łódzkie voivodships, and the lowest - in Podlaskie, Wielkopolskie and Opolskie voivodships.

The greatest Fuzzy Supplementary Indicator values for the deprivation dimensions were noted in the households arrears and unexpected financial expenses dimension, and the lowest in the equipment of households in durables dimension. The hierarchy of voivodships considering fuzzy deprivation incidence with regard to individual dimensions was diversified.

Fuzzy poverty depth in Poland by voivodships

The highest fuzzy poverty depth indicator values were observed in 2007 in Lubelskie, Podkarpackie, Warmińsko-Mazurskie and Dolnośląskie voivodships, while the lowest values were noted in Lubuskie and Mazowieckie voivodships (Table 3). When it comes to the fuzzy deprivation depth, the worst situation was in Łódzkie and Dolnośląskie voivodships, and the best was in Podlaskie, Opolskie and Wielkopolskie.

Voivodships noted the greatest fuzzy poverty deprivation in the household arrears and unexpected financial expenses dimension, and the lowest in the equipment of households in durables dimension. The order of voivodships according to the fuzzy deprivation depth is different in deprivation individual dimensions.

Concluding remarks

The aim of this paper has been to further develop the methodology of multidimensional analysis of poverty by using the fuzzy set theory. In addition to income, other non-monetary indicators were incorporated in the analysis. An important contribution of the paper is the inclusion into poverty analysis, apart from the poverty incidence, also the poverty depth. The study examines the incidence and the depth of monetary and non-monetary poverty in Poland 2007 by voivodships.

The empirical results of the analysis have shown that poverty viewed by income does not coincide with poverty as seen by non-monetary characteristics of this phenomenon. Moreover, the hierarchy of voivodships considering fuzzy deprivation with regard to individual dimensions was diversified. This confirms the hypothesis that poverty in Poland has many dimensions and that its

measurement exclusively form income standing vantage point is highly insufficient. The finding of this research should form a useful source of information in order to implement socio-economic policies to counteract poverty.

REFERENCES

ABEL-SMITH B., TOWNSEND P. (1973): The Poor and the Poorest, in: Atkinson A. B. (eds.) *Wealth, Income and Inequality*, Penguin Education, Harmondsworth.

ATKINSON A. B. AND BOURGUIGNON F. (1982): The Comparison of Multidimensional Distribution of Economic Status, *Review of Economic Studies*, 49: 183–201.

BETTI G., CHELI B., LEMMI A. AND VERMA V. (2006): Multidimensional and Longitudinal Poverty: An Integrated Fuzzy Approach, in: Lemmi A. and Bett G. (eds) *Fuzzy Set Approach to Multidimensional Poverty Measurement*, 111–137, Springer, New York.

BETTI G., VERMA V. (1999): Measuring the Degree of Poverty in a Dynamic and Comparative Context: A Multi-dimensional Approach Using Fuzzy Set Theory, *Proceedings*, ICCS-VI, Vol. 11: 289–301, Lahore, Pakistan, August 27–31, 1999.

BETTI G., VERMA V. (2008): Fuzzy Measures of the Incidence of Relative Poverty and Deprivation: a Multi-dimensional Perspective, *Statistical Methods and Applications*,17: 225–250.

BOURGUIGNON F. AND CHAKRAVARTY S. R. (2003): The Measurement of Multidimensional Poverty, *Journal of Economic Inequality*, 1: 25–49.

CENTRAL STATISTICAL OFFICE (2009): *Incomes and Living Conditions of the Population in Poland. Report from the EU-SILC Survey of 2007 and 2008*, Warsaw.

CERIOLI A., ZANI S. (1990): A fuzzy approach to the measurement of poverty, in: Dagum C. and Zenga M. (eds): *Income and wealth distribution, inequality and poverty*, 272–284, Springer Verlag, Berlin.

CHELI B., LEMMI A. (1995): A Totally Fuzzy and Relative Approach to the Multidimensional Analysis of Poverty, *Economic Notes*, 24: 115–134.

DESAI M., SHAH A. (1988): An Econometric Approach to the Measurement of Poverty, *Oxford Economic Papers*, 40(3), 505–522.

DEUTSCH, J., SILBER, J. (2005): Measuring Multidimensional Poverty: An Empirical Comparison of Various Approaches, *Review of Income and Wealth,* 51(1): 145–74.

Fuzzy Set Approach to Multidimensional Poverty Measurement (2006): Lemmi A. and Betti G. (eds), Springer, New York.

HAGENAARS A. J. M. (1986): *The Perception of Poverty*, North Holland, Amsterdam.

MARSHALL A. (1920): *Principles of Economics*, 8th ed., Macmillan, London.

MCCARTHY P. J., SNOWDEN C. B. (1985): The Bootstrap and Finite Population Sampling, *Vital and Health Statistics*, 2(95), U.S. Government Printing Office, Washington.

PANEK T. (1996): A Multidimensional Analysis of the Poverty in Poland in 1995 and 1996, *Statistics in Transition*, (3)5: 979–1002.

PANEK T. (2009): Poverty Indices in Multidimensional Approach, *Wiadomości Statystyczne* (in Polish), 12: 1–19.

SEN A. K. (1999): *Development as Freedom*, Oxford University Press, Oxford.

TOWNSEND P. (1979): *Poverty in the United Kingdom*, Penguin Books, Middlesex.

TSUI K. Y. (2002): Multidimensional Poverty Indices, *Social Choice and Welfare*, 19(1): 69–93.

WHELAN C. T., LAYTE R., MAITRE B., NOLAN B. (2001): Income, Deprivation and Economic Strain: An Analysis of the European Community Household Panel, *European Sociological Review*, 17: 357–372.

ZADEH L. A. (1965): Fuzzy Sets, *Information and Control,* 8: 338–353.